

# PAINTED CUBES

PATTERNS/FUNCTIONS • GEOMETRY

- Surface area
- Algebra
- Volume
- Organizing data

## Getting Ready

### What You'll Need

Snap Cubes, about 150 per group  
Stick-on dots (optional)

## Overview

Children imagine that cube structures made of Snap Cubes are dipped in paint. They predict how many faces of each Snap Cube would be painted. In this activity, children have the opportunity to:

- ♦ investigate the relationship between volume and surface area of a cube
- ♦ analyze number patterns
- ♦ collect and organize data
- ♦ make predictions and test conjectures



## The Activity

Help children visualize the painted and unpainted parts of the two-cube structure by applying stick-on dots to the painted faces of each cube. Then, when you separate the cubes, it will be obvious which faces are not painted.

## Introducing

- ♦ Ask children to imagine that you have dipped one white Snap Cube into a special, quick-drying red paint. Explain that all six faces would be red.
- ♦ Then show children two white cubes snapped together. Ask children to describe which faces of the cubes would be red if the two-cube structure were dipped in the red paint. Take apart the structure to point out that only five faces of each cube would be red.

## On Their Own

*How many faces of each Snap Cube get painted when a cube structure is dipped in paint?*

- Work with a group. Imagine that you have to dip bigger and bigger cube structures into quick-drying paint.
- Use Snap Cubes to build each cube structure starting with a  $2 \times 2 \times 2$  cube.
- For each structure, figure out how many Snap Cubes would have 3 faces painted, 2 faces painted, 1 face painted, and 0 faces painted.
- When you build your cube structures, you may want to use 1 color for Snap Cubes with 1 face painted, another color for the Snap Cubes with 2 faces painted, and so on.
- Record your findings and look for patterns.
- Continue investigating until you can figure out what would happen to a cube structure of any size.

## The Bigger Picture

### *Thinking and Sharing*

Ask children how they organized their data and what number patterns they found. Then, create a class chart with the headings *Dimensions of Cube*, *# of Snap Cubes*, *3 faces painted*, *2 faces painted*, *1 face painted*, and *0 faces painted*.

Use prompts such as these to promote class discussion:

- ◆ How did you organize your data?
- ◆ What patterns did you find?
- ◆ How do these number patterns relate to what is happening with each cube structure?
- ◆ Could the number 708 appear in the column labeled *2 faces*? Why or why not?
- ◆ Could the number 1,000 appear in the column labeled *1 face*? Why or why not?
- ◆ What predictions could you make about a cube structure that has 12 cubes on each edge? How did you arrive at these predictions?

### *Extending the Activity*

Have children repeat the activity for single-layer prisms, starting with a  $1 \times 1 \times 2$  rectangular prism made from two Snap Cubes. They should increase the size of the prism by one unit in length and one in width for each term, that is,  $1 \times 2 \times 3$ ,  $1 \times 3 \times 4$ ,  $1 \times 4 \times 5$ , and so on.

Children’s records of the data may look like this:

Dimensions of Cube	# of Snap Cubes	# of Cubes Painted			
		3 faces	2 faces	1 face	0 faces
2 x 2 x 2	8	8	0	0	0
3 x 3 x 3	27	8	12	6	1
4 x 4 x 4	64	8	24	24	8
5 x 5 x 5	125	8	36	54	27
6 x 6 x 6	216	8	48	96	64

As the class analyzes the data, be sure they can see the connections between the numbers on the chart and the actual physical models. It may be helpful to have color-coded models at the front of the class to use during discussion. The Snap Cubes with 1 face painted would be one color, those with 2 faces painted would be another color, and so on.

Looking at their structures and the data in their charts, children are likely to point out that the number of Snap Cubes needed for each cube structure is the product of the dimensions of the cube structure. Although this is not the focus of the activity, this information helps children to check their work since the sum of the painted and unpainted Snap Cubes must equal the total number of cubes in the structure. Some children may point out that this total number of cubes also represents the volume of the structure.

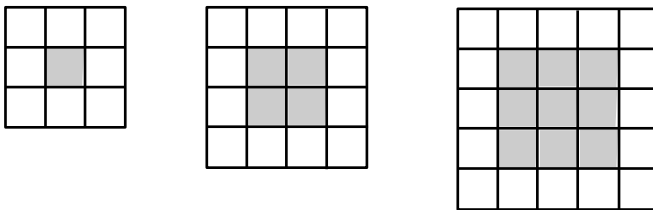
Children will notice that there are always 8 Snap Cubes with three painted faces. These are the Snap Cubes at the vertices of the cube structure. Children can easily verify that every cube, no matter how large, has only 8 vertices.

The Snap Cubes that get paint on two faces are the ones that are along the edges and between the vertices. In a 3 x 3 x 3 cube, there is one such Snap Cube along each of the 12 edges of the cube structure, and so there are 12 cubes with two faces painted. Likewise, in a 4 x 4 x 4 cube structure, there are two such Snap Cubes along each of the 12 edges of the cube structure, and so there are 24 cubes with two faces painted. Since every cube, no matter how large, has 12 edges, the cubes with two faces painted will always be multiples of 12. To find the specific number for a given cube structure, subtract 2 vertex cubes from the number of cubes along the edge of the

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structure and then multiply that answer by 12. Once children discover this pattern, they should be able to conclude that is possible to have 708 in the *2 faces* column because 708 is a multiple of 12.

The Snap Cubes that have 1 face painted are those that are on the faces of the structure but are not at the vertices nor along the edges. Children will probably recognize that the numbers of Snap Cubes with 1 face painted are multiples of 6 because every cube, no matter how large, has exactly six faces. Since 1,000 is not a multiple of 6, children should be able to conclude that there is no cube structure with 1,000 in the column labeled *1 face*. If children have color-coded their structures, they will notice that the Snap Cubes with 1 face painted form a square with dimensions 2 less than the face of the cube.



Children may be able to come up with a generalization for finding the number of Snap Cubes with one face painted: *Subtract 2 from the dimension of the cube and square that answer. Then you multiply by 6 because there are 6 faces on a cube.*

The Snap Cubes in the structure that don't need any paint will be "hidden" in the center. The number of Snap Cubes with 0 painted faces is always the number of Snap Cubes in the cube structure that has dimensions 2 less than the one being described. For example, a  $4 \times 4 \times 4$  cubes has 64 Snap Cubes. Inside, there is a  $2 \times 2 \times 2$  cube that no one can see. And so,  $2 \times 2 \times 2$  or 8 Snap Cubes have no painted faces.

This activity provides children with practice in visualizing and finding patterns and lays the foundation for future work with algebraic functions and variables.

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